

On the Relation Between Spotless Days and the Sunspot Cycle

Robert M. Wilson and David H. Hathaway

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Space Administration

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LIST OF ACRONYMS AND ABBREVIATIONS

AM	ascent-maximum amplitude (class)
AP	ascent-period (class)
ASC	ascent duration
E(FSD)	epoch of first spotless day
E(LSD)	epoch of last spotless day
EM	epoch of maximum amplitude
Em	epoch of minimum amplitude
FL	fast rise-large amplitude
fl	fast rise-long period
FS	fast rise-small amplitude
fs	fast rise-short period
FSD	first spotless day
LSD	last spotless day
NSD	number of spotless days
NSD(ASC)	number of spotless days during ASC
NSD(t_1)	number of spotless days during t_1
PER	period
SL	slow rise-large amplitude
sl	slow rise-long period
SS	slow rise-small amplitude
ss	slow rise-short period

NOMENCLATURE

cl	confidence level
n	number
P	probability
r	correlation coefficient
r^2	coefficient of determination
RM	maximum amplitude
Rm	minimum amplitude
$(Rm)_{n+1}$	Rm for cycle $n+1$
se	standard error of estimate
t	elapsed time in months from Em
t_1	elapsed time in months from E(FSD) to Em
$(t_1)_{n+1}$	t_1 for cycle $n+1$
t_2	elapsed time in months from E(LSD) to EM
t_3	elapsed time in months from E(FSD) to E(LSD)
t_4	elapsed time in months from E(LSD) for cycle n to E(FSD) for cycle $n+1$
$(t_4)_n$	t_4 for cycle n
x	independent variable
y	dependent variable

TECHNICAL PUBLICATION

ON THE RELATION BETWEEN SPOTLESS DAYS AND THE SUNSPOT CYCLE

1. INTRODUCTION

In the mid-19th century (1826–1868), a German apothecary and amateur astronomer, Samuel Heinrich Schwabe, studied the Sun’s annual variation of the number of clusters of spots and the number of days when no spots were observed. From his observations, he found that the Sun’s spottedness varies over an interval of ≈ 10 yr, with the peak in the number of clusters of spots indicating a maximum in solar activity and the peak in the number of days when no spots were observed indicating a minimum in solar activity. Thus, he asserted that solar activity, as evinced by sunspots, varies somewhat regularly over time in cyclic fashion.^{1–5}

Following this startling declaration, the professional Swiss astronomer Rudolf Wolf devised a simple formula for describing the solar cycle, one that combines the number of groups—similar to Schwabe’s clusters of spots—and the actual number of individual spots that can be seen on the Sun each day. On the basis of his relative sunspot number, he was able to approximate the past record of solar activity from the days of Galileo Galilei (1610) and initiate an international collaboration for monitoring sunspots that continues through the present. In contrast to the 10-yr length found by Schwabe, Wolf found the average length of the solar cycle to be ≈ 11 yr, although, strictly speaking, individual cycles were seen to vary in length up to several years either side of the 11-yr average.^{5–8}

More recently, on the basis of another proxy of solar activity—the group sunspot number, arguments have been raised that, while the two proxies of solar activity differ only slightly since about 1882, their differences are more substantial during earlier years. Hence, because the group sunspot number is based on a greater number of observers than was used by Wolf, it has been suggested that Wolf’s reconstruction—both in terms of timing and amplitude—might be in error, especially for the earliest cycles—those prior to the mid-1800s.^{9–16}

About 10 yr ago, Wilson showed that the first spotless day (FSD) prior to the onset of the new cycle, which occurs during the declining phase of the old cycle, can be used for the prediction of the occurrence of solar minimum of the new cycle.¹⁷ In this Technical Publication, the use of spotless days in relation to the timing and size of the solar cycle is again examined, but in much greater detail, as part of a continuing study of the characteristics of sunspot cycles.^{18–55}

2. RESULTS AND DISCUSSION

2.1 Characteristics of Spotless Days

Figure 1 (bottom) displays the variation of smoothed monthly mean sunspot number (the 12-mo moving average of monthly mean sunspot number) for the interval of January 1973 through January 2004, plotted as the dotted line. Individual sunspot cycle numbers (21–23) are identified along the bottom, as are the epochs of cycle minimum (Em) and maximum (EM). Conventionally, a sunspot cycle is reckoned using smoothed monthly mean sunspot number for determining its minimum and maximum, ascent and descent durations, and period (or cycle length). In the upper portion of figure 1, the number of spotless days (NSD) is plotted, which allows for easy identification of the occurrences of the FSD and the last spotless day (LSD) for each of cycles 21–23. The actual number of spotless days that were observed in each of the cycles is also shown. Depicted between the two timelines are various descriptors based on the occurrence of spotless days. These include t_1 , t_2 , t_3 , and t_4 , where t_1 is the elapsed time from the epoch of FSD to Em, t_2 is the elapsed time from the epoch of LSD to EM, t_3 is the elapsed time from the epoch of FSD to the epoch of LSD, and t_4 is the elapsed time from the epoch of LSD for cycle n to the epoch of FSD for cycle $n + 1$.

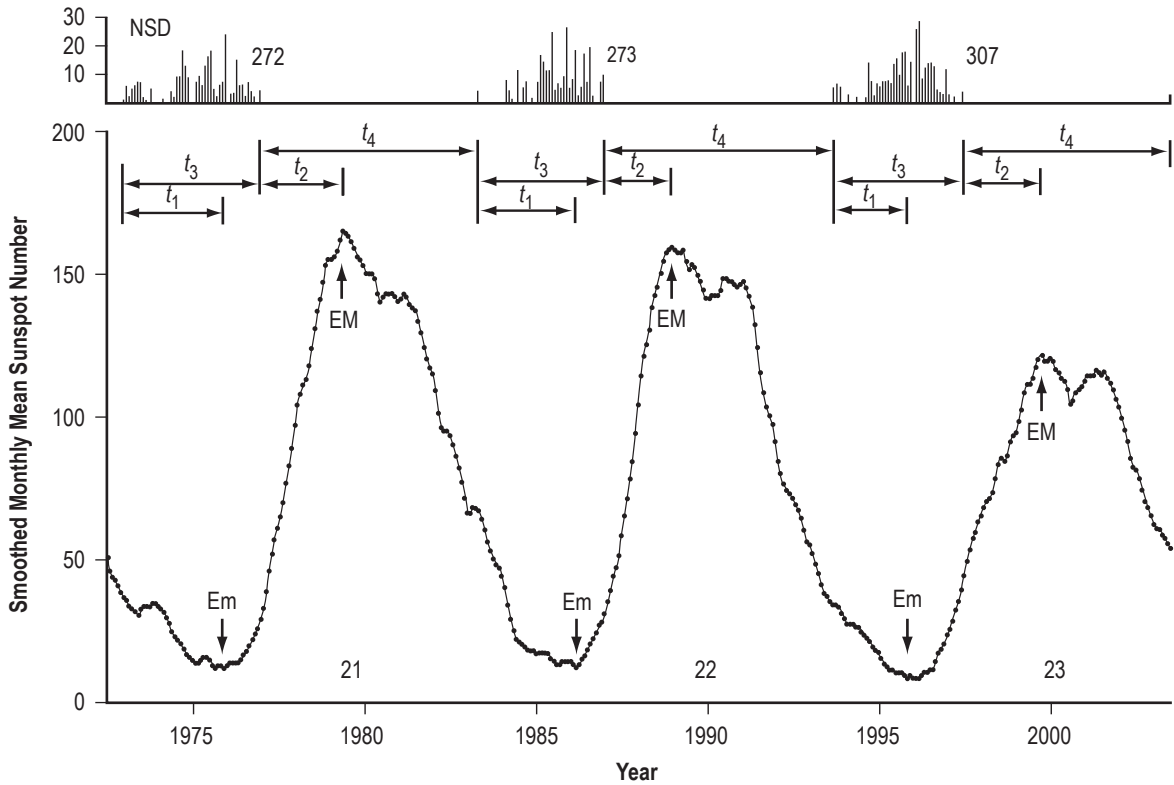


Figure 1. Variation of smoothed monthly mean sunspot number and spotless days for January 1973 through January 2004.

Table 1 summarizes selected parameters for the modern era sunspot cycles 10–24, including some information on cycle 9. Identified in the table are the cycle number, the epochs of FSD, minimum, LSD, and maximum—E(FSD), Em, E(LSD), and EM; t_1 , t_2 , t_3 , and t_4 ; ascent duration (ASC) and period (PER); the number of spotless days (NSD) during t_1 , t_3 , and ASC; the minimum and maximum amplitudes R_m and RM ; and the ascent-period (AP) and ascent-maximum amplitude (AM) classifications of the cycles based on their median values, where sl, fl, ss, and fs mean, respectively, slow rise-long period, fast rise-long period, slow rise-short period, and fast rise-short period for the AP class, and SL, FL, SS, and FS mean, respectively, slow rise-large amplitude, fast rise-large amplitude, slow rise-small amplitude, and fast rise-small amplitude for the AM class.

Table 1. Selected sunspot cycle parameters.

Cycle	E(FSD)	Em	E(LSD)	EM							NSD					Class	
					t_1 (mo)	t_2 (mo)	t_3 (mo)	t_4 (mo)	ASC (mo)	PER (mo)	t_1 (mo)	t_3 (mo)	ASC (mo)	R_m	RM	AP	AM
9	–	Jul 1843	–	Feb 1848	–	–	–	–	55	149	–	–	–	10.5	131.6	sl	SL
10	May 1849	Dec 1855	Apr 1858	Feb 1860	79	22	107	42	50	135	296	654	358	3.2	97.9	sl	SS
11	Oct 1861	Mar 1867	Jul 1869	Aug 1870	65	13	93	46	41	141	202	405	203	5.2	140.5	fl	FL
12	May 1873	Dec 1878	Sep 1883	Dec 1883	67	3	124	16	60	135	737	1,010	273	2.2	74.6	sl	SS
13	Jan 1885	Mar 1890	Dec 1891	Jan 1894	62	25	83	47	46	142	580	734	154	5	87.9	fl	FS
14	Nov 1895	Jan 1902	Jul 1905	Feb 1906	74	7	116	15	49	139	628	934	306	2.6	64.2	sl	SS
15	Oct 1906	Aug 1913	Oct 1916	Aug 1917	82	10	120	42	48	120	724	1,021	297	1.5	105.4	ss	SS
16	Apr 1920	Aug 1923	Jul 1926	Apr 1928	40	21	75	50	56	121	316	531	215	5.6	78.1	ss	SS
17	Sep 1930	Sep 1933	Jul 1935	Apr 1937	36	21	58	76	43	125	298	568	270	3.4	119.2	fs	FL
18	Nov 1941	Feb 1944	Sep 1945	May 1947	27	20	46	63	39	122	114	268	154	7.7	151.8	fs	FL
19	Dec 1950	Apr 1954	Oct 1955	Mar 1958	40	29	58	73	47	126	226	445	219	3.4	201.3	fs	FL
20	Nov 1961	Oct 1964	Aug 1966	Nov 1968	35	27	57	83	49	140	117	226	109	9.6	110.6	sl	SS
21	Jul 1973	Jun 1976	Jul 1977	Dec 1979	35	29	48	76	42	123	189	272	83	12.2	164.5	fs	FL
22	Nov 1983	Sep 1986	Jul 1987	Jul 1989	34	24	44	81	34	116	187	273	86	12.3	158.5	fs	FL
23	Apr 1994	May 1996	Jan 1998	Apr 2000	25	27	45	72	47	–	134	307	173	8	120.8	f?*	FL
24	Jan 2004	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

t_1 : E(FSD) \longrightarrow Em
 t_2 : E(LSD) \longrightarrow EM
 t_3 : E(FSD) \longrightarrow E(LSD)
 t_4 : E(LSD) $_n$ \longrightarrow E(FSD) $_{n+1}$

* Period has not, as yet, been strictly determined.

Figures 2 and 3, respectively, show the 2×2 contingency tables for the AP and AM classes. Clearly, of the two classification schemes, the AM classification is presently the only statistically significant grouping, having a probability of obtaining the observed result, or one more suggestive of a departure from independence, based on Fisher's exact test,⁵⁶ $P=0.9$ percent. Thus, there appears to exist a real correlation between the length of rise of a cycle and its maximum amplitude, associating fast-rising cycles with larger than average sized cycles and slow-rising cycles with smaller sized cycles. This correlation is often referred to as the “Waldmeier effect,” named in honor of Max Waldmeier, a former director of the Swiss Federal Observatory in Zürich, Switzerland, who first suggested such a relationship. (Interestingly, if cycle 23 turns out to be a cycle of shorter length, then it would be classified as fs and the probability would be reduced from 14.3 percent to ≈ 10 percent, which indicates a marginally significant result for the AP classification scheme.)

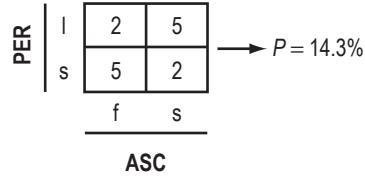


Figure 2. Ascent-period class of sunspot cycles for cycles 9–22.

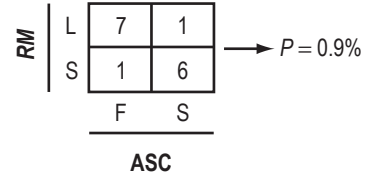


Figure 3. Ascent-maximum amplitude class of sunspot cycles for cycles 9–23.

2.2 Cyclic Variations

Figure 4 depicts the cyclic variation of Rm , RM , ASC , and PER for the modern era cycles. In figure 4 and succeeding figures, except figs. 13 and 15, filled triangles indicate cycles of shorter length and filled circles indicate cycles of longer length. The filled box, associated with cycle 23, merely indicates that its period class is presently unknown.

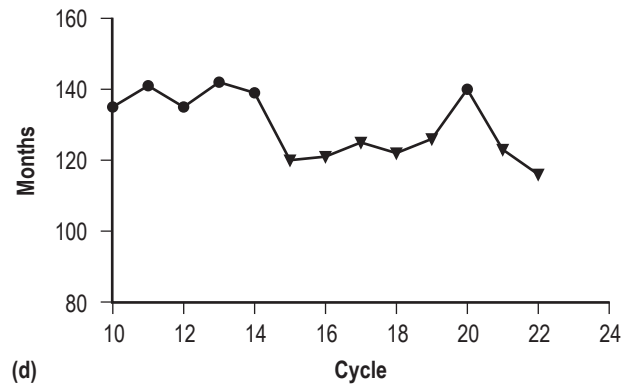
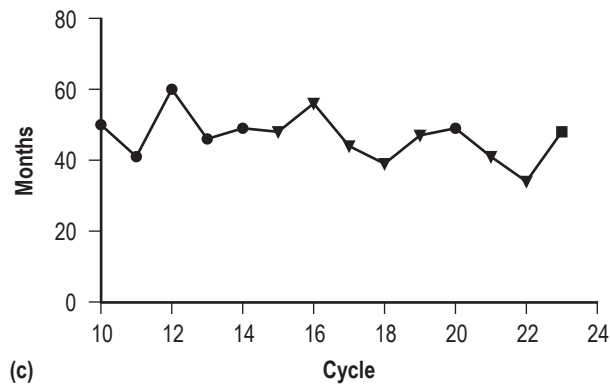
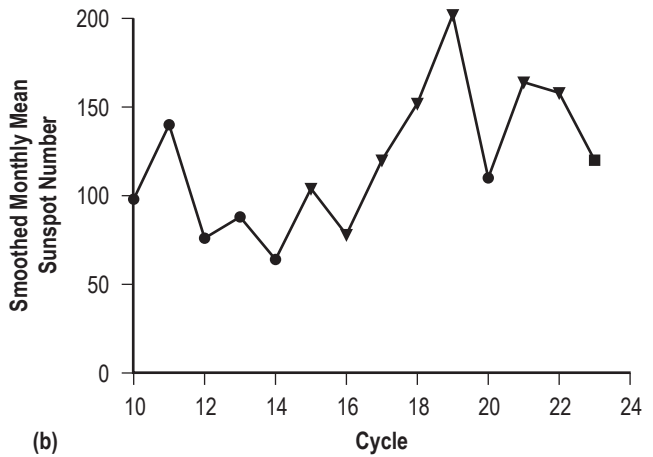
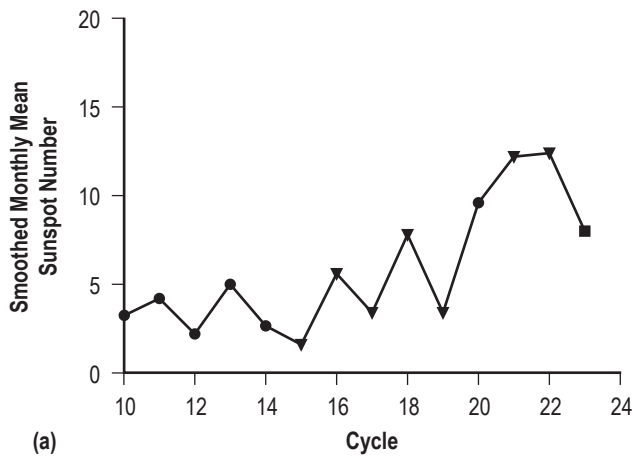


Figure 4. Variation of (a) Rm , (b) RM , (c) ASC , and (d) PER for cycles 10–23.

On average, the elapsed time from cycle minimum to succeeding cycle minimum is ≈ 131 mo, having a standard deviation of ≈ 10.3 mo. Close inspection of PER, however, reveals that seven of the last eight fully described sunspot cycles have been cycles of shorter period, averaging ≈ 121.9 mo and having a standard deviation of 3.3 mo. Thus, statistically speaking, if cycle 23 turns out to be a cycle of shorter period, then Em for cycle 24 would be expected to occur about August 2006 ± 8 mo. On the other hand, if cycle 23 bucks this recent trend and turns out to be a cycle of longer period, averaging ≈ 138.7 mo with a standard deviation of 3 mo, then Em for cycle 24 would be delayed until about late 2007 to early 2008.

Interestingly, if cycle 23 turns out to be a cycle of shorter period, then both Rm and RM would be expected to be larger than average in size, since shorter period cycles often are followed by cycles of larger than average minimum and maximum amplitudes. From figure 4, it can be shown that there exists a statistically significant (at the 0.5-percent level of significance or 99.5-percent level of confidence) upward trend in Rm , one that can be described linearly as $y = -4.416 + 0.622x$, where y is Rm and x is the cycle number. The inferred regression has a coefficient of correlation $r = 0.726$ and standard error of estimate (se) of 2.6 units of sunspot number. Thus, presuming that the upward trend in Rm continues, cycle 24 would be expected to have an $Rm = 10.5 \pm 4.6$ —the 90-percent prediction interval.

On the basis of figure 4, evidence for a strong upward linear trend in RM is lacking. However, as shown in a previous study using group sunspot number,¹⁶ the trend is quite noticeable, suggesting an RM that measures $\approx 136.5 \pm 41.3$ —the 90-percent prediction interval—for cycle 24. Also, because of the Waldmeier effect, a larger than average sized RM for cycle 24 implies that its ASC will be shorter than average, meaning that cycle 24 would be expected to be a fast-rising cycle, one that peaks in <4 yr after Em, or probably sometime in 2010.

Figure 5 displays the variation of t_1 , t_2 , t_3 , and t_4 . As before for Rm and RM , variations of these parameters also appear to vary systematically, with a substantial decrease being seen in t_1 and t_3 and a substantial increase being seen in t_2 and t_4 .

For t_1 , eight of the last eight cycles (16–23) have had a value that has ranged between 25 and 40 mo, averaging ≈ 34 mo with a standard deviation of 5.5 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, Em for cycle 24 should occur about November 2006 ± 10 mo, using January 2004 as E(FSD), for cycle 24, a date in close agreement found previously based on the assumption that cycle 23 is a cycle of shorter period.

For t_2 , eight of the last eight cycles (16–23) have had a value that has ranged between 20 and 29 mo, averaging ≈ 25 mo with a standard deviation of 3.7 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, EM for cycle 24 should follow E(LSD) by 2 yr. E(LSD) for cycle 24 has not yet been observed.

For t_3 , eight of the last eight cycles (16–23) have had a value that has ranged between 44 and 75 mo, averaging ≈ 54 mo with a standard deviation of 10 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, the interval between E(FSD) and E(LSD) for cycle 24 should extend ≈ 4.5 yr, inferring that E(LSD) for cycle 24 should occur sometime in 2008.

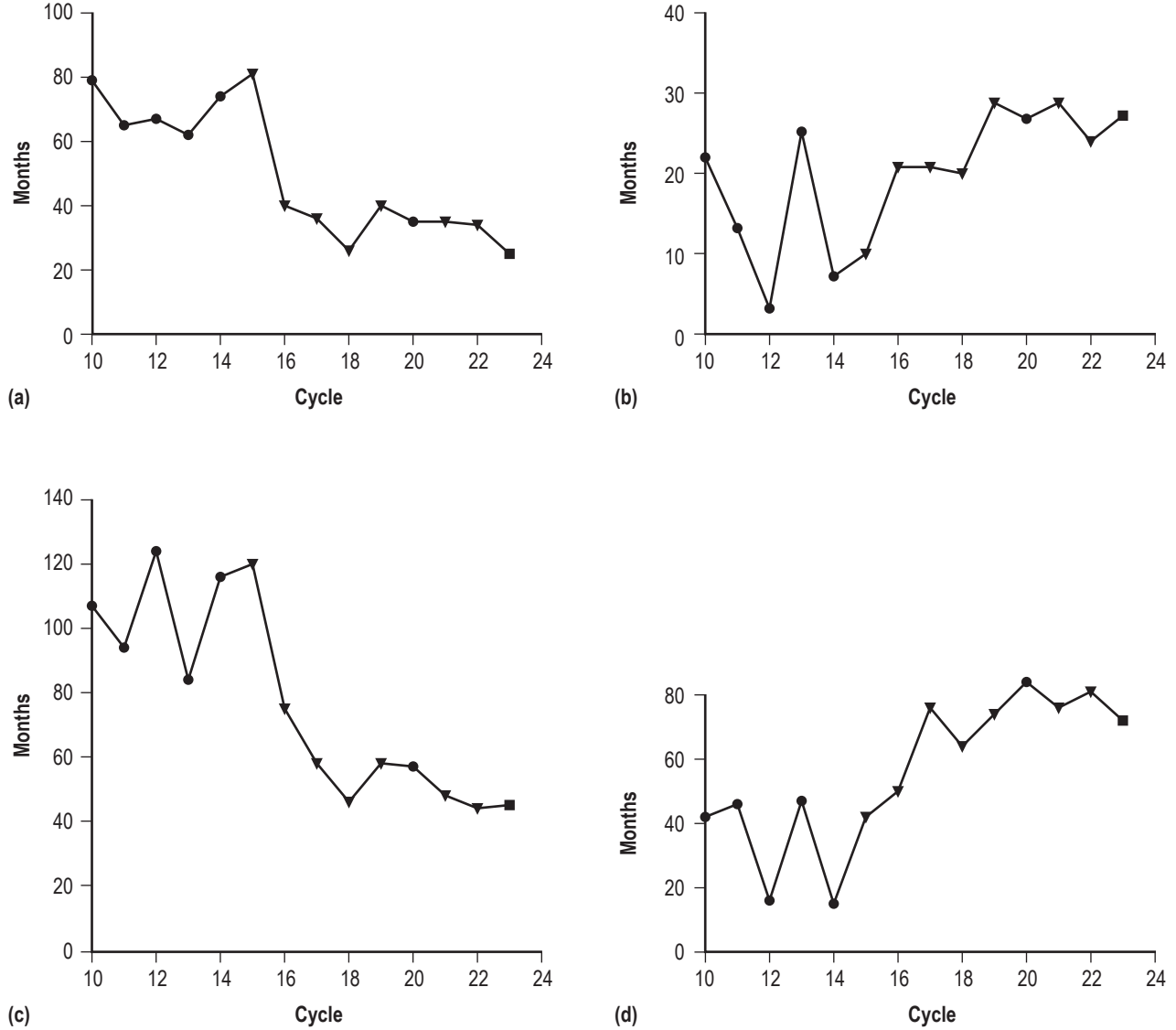


Figure 5. Variation of (a) t_1 , (b) t_2 , (c) t_3 , and (d) t_4 for cycles 10–23.

For t_4 , eight of the last eight cycles (16–23) have had a value that has ranged between 50 and 83 mo, averaging ≈ 72 mo with a standard deviation of 11 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, the interval between E(LSD) of cycle 24 and E(FSD) of cycle 25 should extend ≈ 6 yr, inferring that E(FSD) for cycle 25 should occur sometime around 2014.

Figure 6 shows the variation of the elapsed time from EM for cycle n to E(FSD) for cycle $n+1$, mathematically equal to $t_4 - t_2$. For the last eight cycles (16–23), this difference has spanned 29 to 57 mo, averaging ≈ 47 mo with a standard deviation of 9 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, the time from EM for cycle 24 to E(FSD) for cycle 25 should be ≈ 4 yr. During this interval, there will be zero spotless days; this interval corresponds to about 2010–2013.

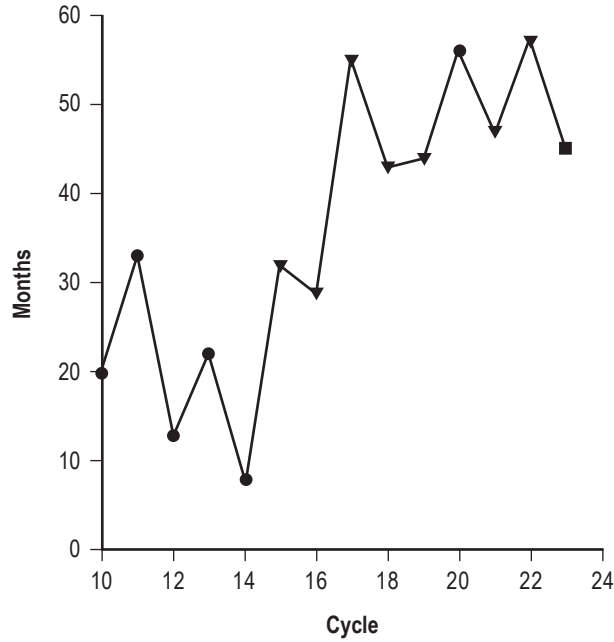


Figure 6. Variation of $t_4 - t_2$ for cycles 10–23.

Figure 7 displays the variation of $t_3 + t_4$, which is the elapsed time from E(FSD) for cycle n to E(FSD) for cycle $n + 1$. Notice that a statistically significant downward trend is hinted, with the largest discrepancy being associated with cycle 15, the cycle marking the start of a string of short-period cycles. For cycle 24, the regression predicts the value for $t_3 + t_4$ to be 119.5 ± 20.5 mo—the 90-percent prediction interval.

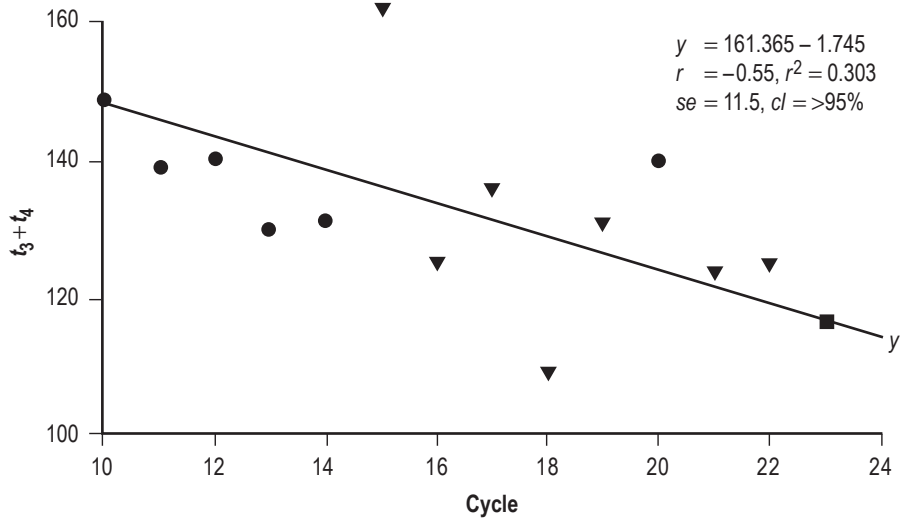


Figure 7. Variation of $t_3 + t_4$ for cycles 10–23.

Figures 8 and 9 depict the NSDs for three intervals of time: t_1 , t_3 , and ASC. For t_1 , eight of the last eight cycles (16–23) have had a value of NSD that spans 114 to 316 days, averaging ≈ 198 days with a standard deviation of 78 days. For t_3 , eight of the last eight cycles (16–23) have had a value of NSD that spans 226 to 568 days, averaging ≈ 361 days with a standard deviation of 133 days. More interestingly, for ASC, there appears to exist a statistically significant downward trend in NSD ($r = -0.7$ and $se = 63.8$), such that during the rising portion of cycle 24, one should expect < 215 spotless days (100 ± 114 days).

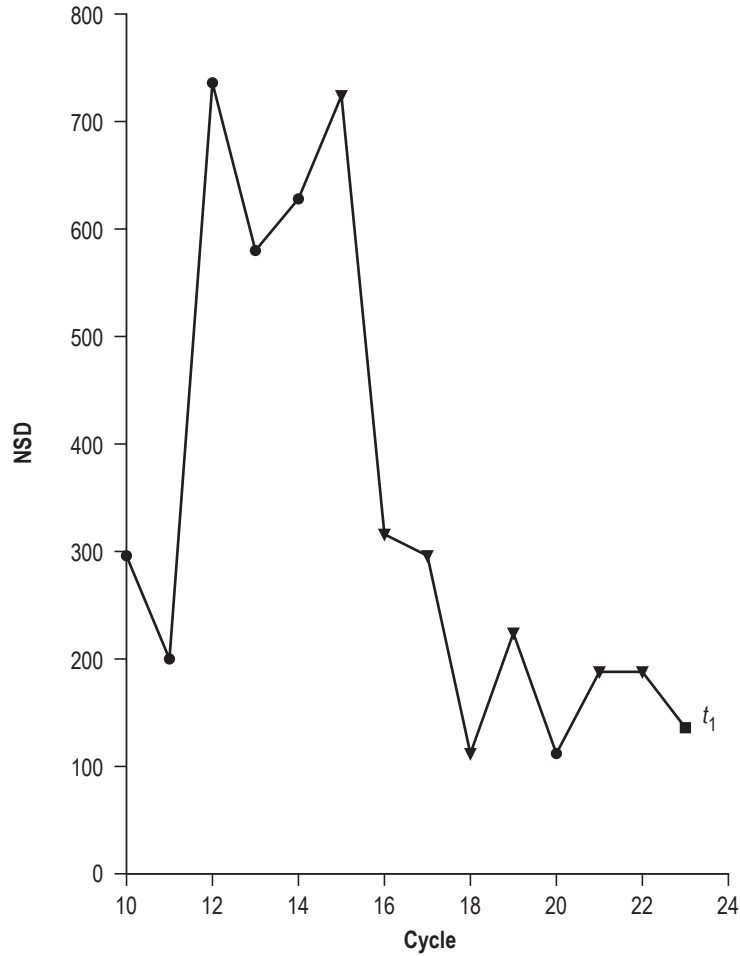


Figure 8. Variation of NSD during t_1 for cycles 10–23.

2.3 Parametric Correlations

Figure 10 shows scatterplots of t_2 , t_3 , and t_4 against t_1 . Expressed as linear regressions, all are found to be statistically significant; expressed as 2×2 contingency tables, only the latter two scatterplots are statistically meaningful. Thus, knowledge of t_1 allows for determination of the later occurring parameters t_2 , t_3 , and t_4 . Furthermore, knowledge of t_1 seems to provide a strong indication for the period class of a sunspot cycle, with shorter period usually being associated with shorter t_1 (≤ 40 mo) and longer period usually being associated with longer t_1 (≥ 62 mo). The recent behavior of t_1 (cycles 16–23) has

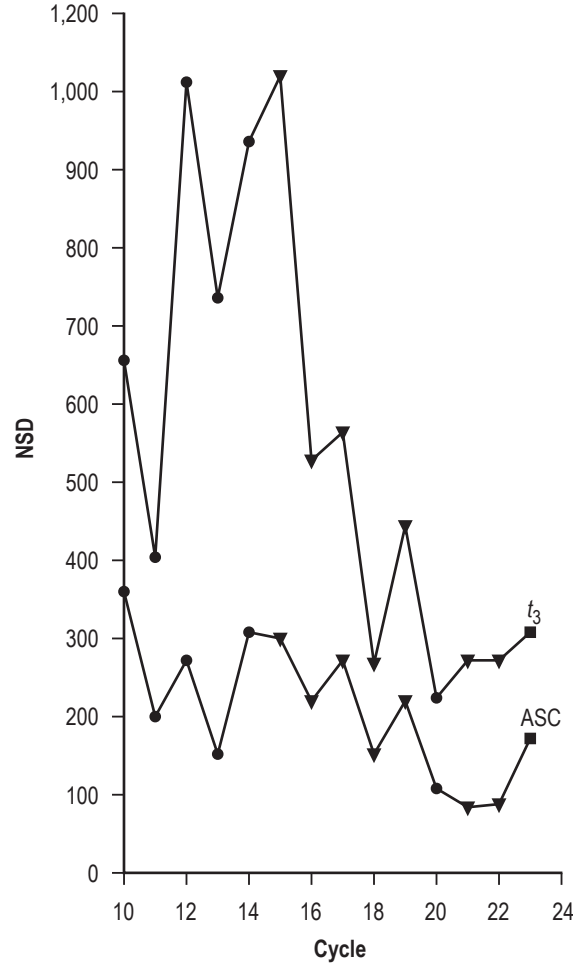


Figure 9. Variation of NSD during t_3 and ASC for cycles 10–23.

consistently been of shorter length and six of seven of the fully described cycles have been of shorter period. (Recall that, as yet, cycle 23's period remains unknown.) Presuming cycle 24 to have the average length of 34 mo for t_1 , based on cycles 16–23, then t_2 , t_3 , and t_4 equals, respectively, 24 ± 11 mo, 54 ± 18 mo, and 70 ± 24 mo using the inferred regressions—the 90-percent prediction intervals.

Figure 11 displays the scatterplots of t_2 and t_4 against t_3 . Expressed as linear regressions, both are found to be statistically significant; expressed as 2×2 contingency tables, only the latter scatterplot is statistically meaningful. Thus, knowledge of t_3 allows for determination of the later occurring parameters t_2 and t_4 . Presuming cycle 24 to have the average length of 54 mo for t_3 , based on cycles 16–23, then t_2 and t_4 equals, respectively, 25 ± 9 mo and 71 ± 17 mo—the 90-percent prediction intervals.

Figure 12 depicts the scatterplot of t_4 against t_2 . Expressed as a linear regression, it is found to be statistically significant; expressed as a 2×2 contingency table, it is not. Thus, knowledge of t_2 allows for determination of the later occurring parameter t_4 . Presuming cycle 24 to have the average length of 25 mo for t_2 , based on cycles 16–23, then t_4 equals 68 ± 22 mo—the 90-percent prediction interval.

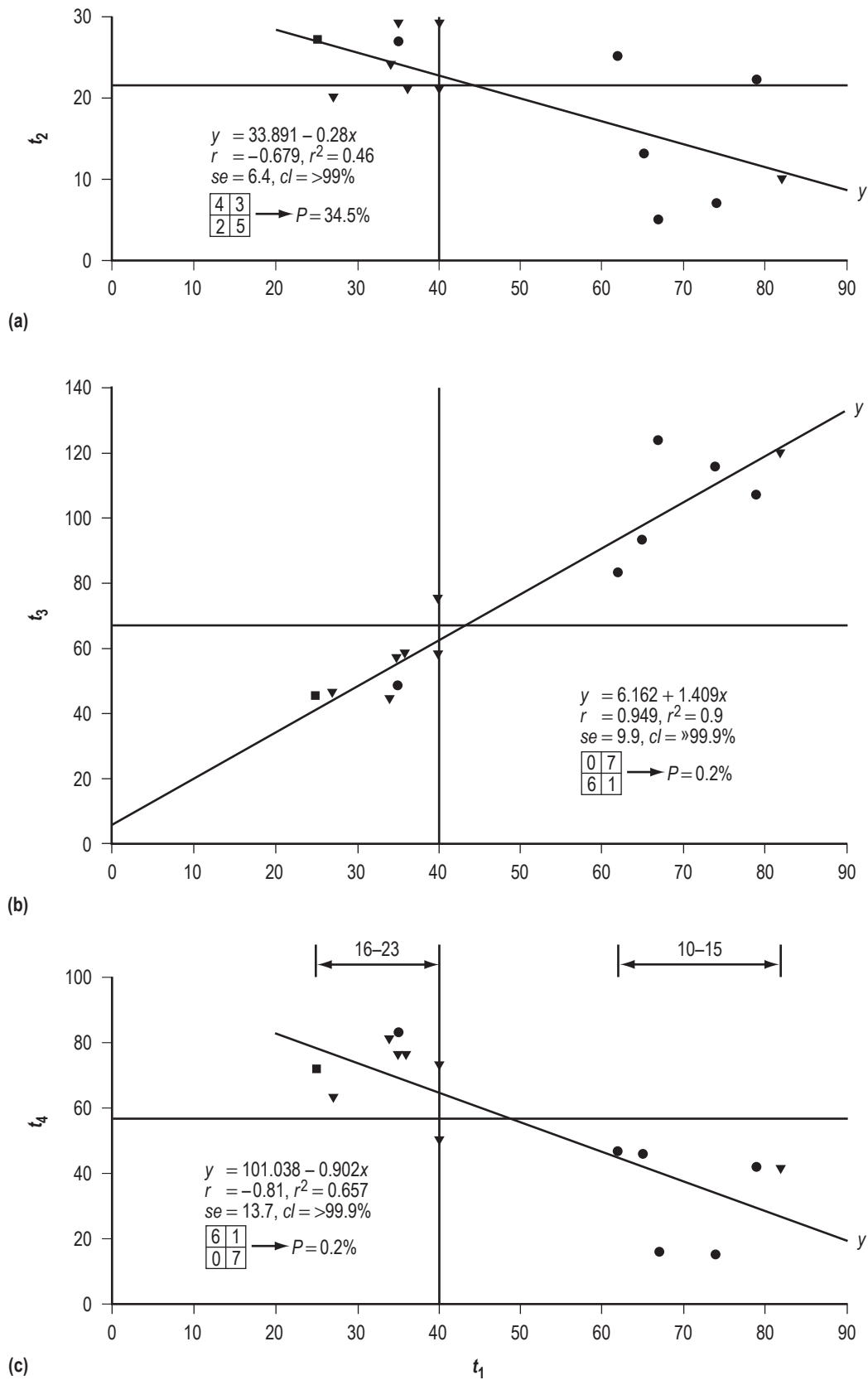


Figure 10. Scatterplots of (a) t_2 , (b) t_3 , and (c) t_4 versus t_1 .

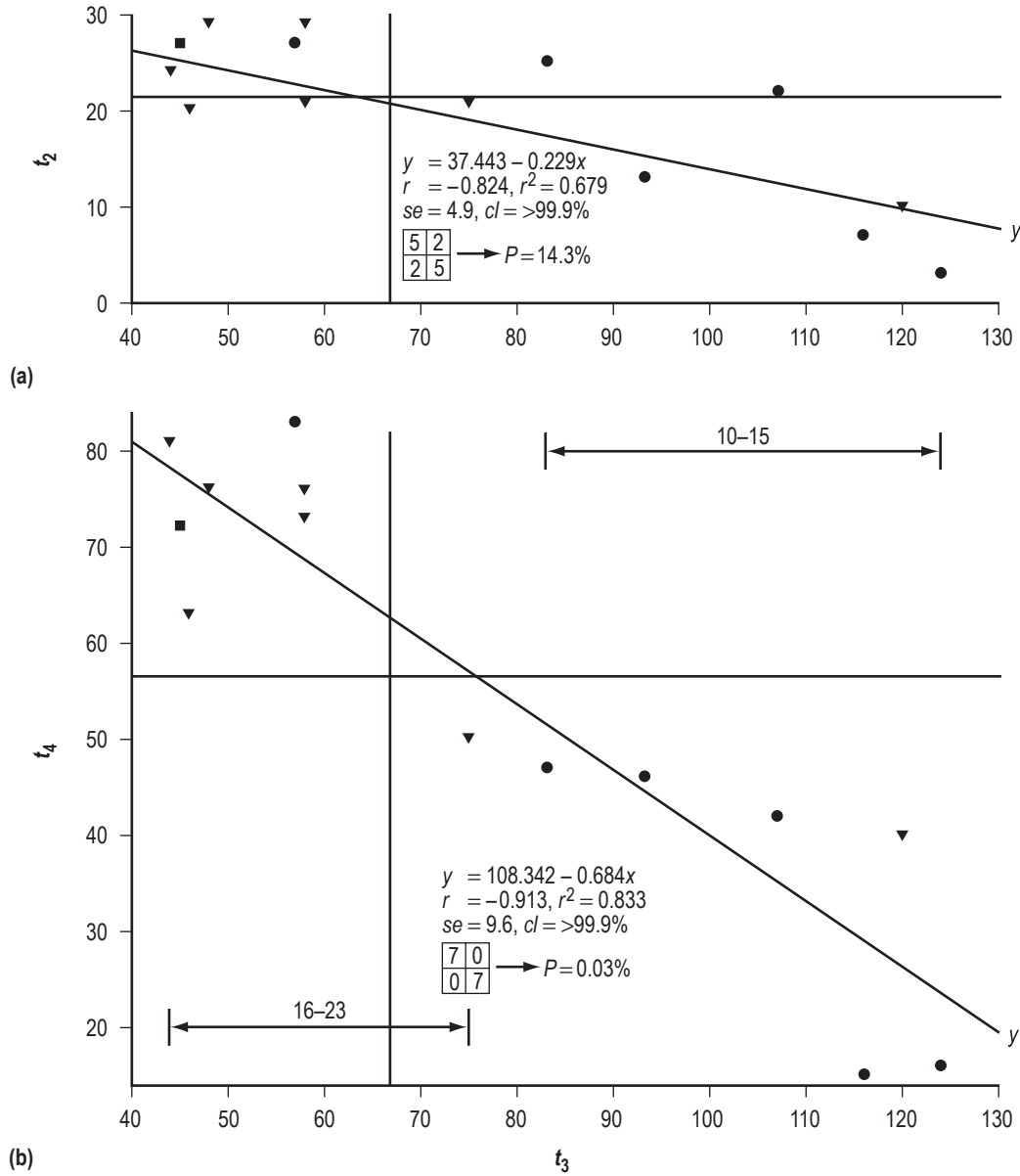


Figure 11. Scatterplots of (a) t_2 and (b) t_4 versus t_3 .

Figure 13 shows the scatterplot of t_1 for cycle $n+1$ against t_4 for cycle n . The correlation between the parameters is statistically important, whether it is expressed linearly or as a 2×2 contingency table. Furthermore, it is a valuable correlation that is directly applicable for predicting t_1 for cycle 24, since t_4 for cycle 23 has been determined (equal to 72 mo, on the basis of January 2004 being the E(FSD) for cycle 24; see table 1), indicated by the downward-pointing arrow along the x axis. Based on the linear fit, t_1 for cycle 24 is expected to be $\approx 36 \pm 21$ mo—the 90-percent prediction interval. Based on the 2×2 contingency table, because t_4 for cycle 23 is to the right of the median ($=50$ mo, the vertical line), the expected value for t_1 for cycle 24 is expected to fall in the lower right quadrant of the 2×2 contingency table, indicating that its value should be <40 mo (the horizontal line), probably somewhere between 25 and 36 mo—the observed range of values for the lower right quadrant. Using January 2004 as

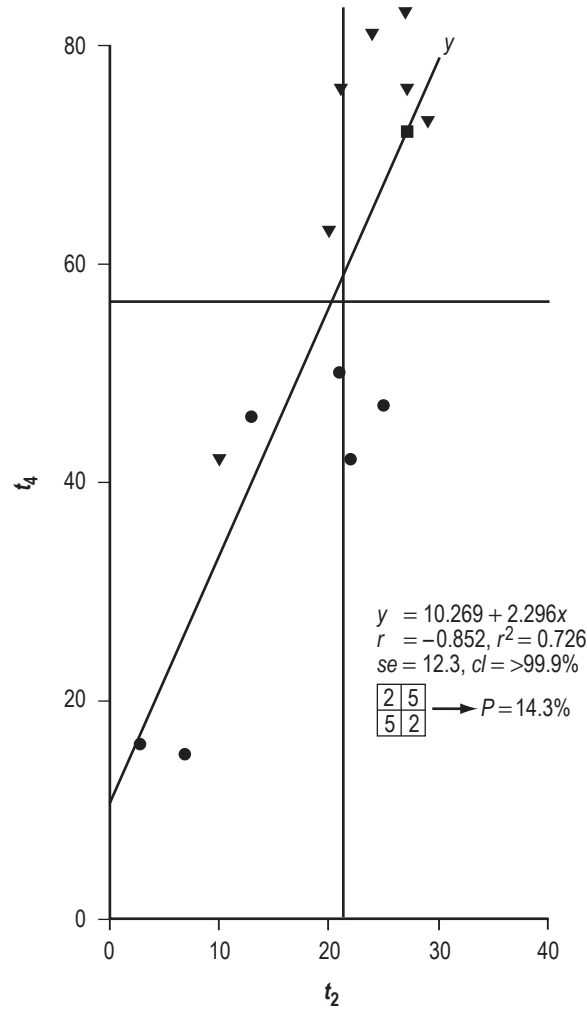


Figure 12. Scatterplot of t_4 versus t_2 .

E(FSD) for cycle 24 and expecting t_1 for cycle 24 to measure about 25–36 mo, based on the 2×2 contingency table, it follows that Em for cycle 24 should occur sometime in 2006 and implies that cycle 23 is a cycle of shorter period.

Figure 14 displays the scatterplot of Rm against t_1 . The correlation, whether expressed linearly or as a 2×2 contingency table, is statistically important. Based on the linear fit and using t_1 equal to 34 mo (the average of t_1 values for cycles 16–23), Rm for cycle 24 is expected to be $\approx 7.9 \pm 4.6$ —the 90-percent prediction interval. Based on the 2×2 contingency table, because t_1 for cycle 24 is expected to be < 40 mo, Rm for cycle 24 should be in the upper left quadrant of the table, inferring that its value will be larger than ≈ 5.4 .

Figure 15 depicts the scatterplot of Rm for cycle $n + 1$ against t_4 for cycle n . Expressed as a linear fit, the correlation is statistically important. Since t_4 has been determined to be ≈ 72 mo for cycle 23, denoted by the downward-pointing arrow along the x axis, Rm for cycle 24 should measure $\approx 8.1 \pm 4.7$ —the 90-percent prediction interval.

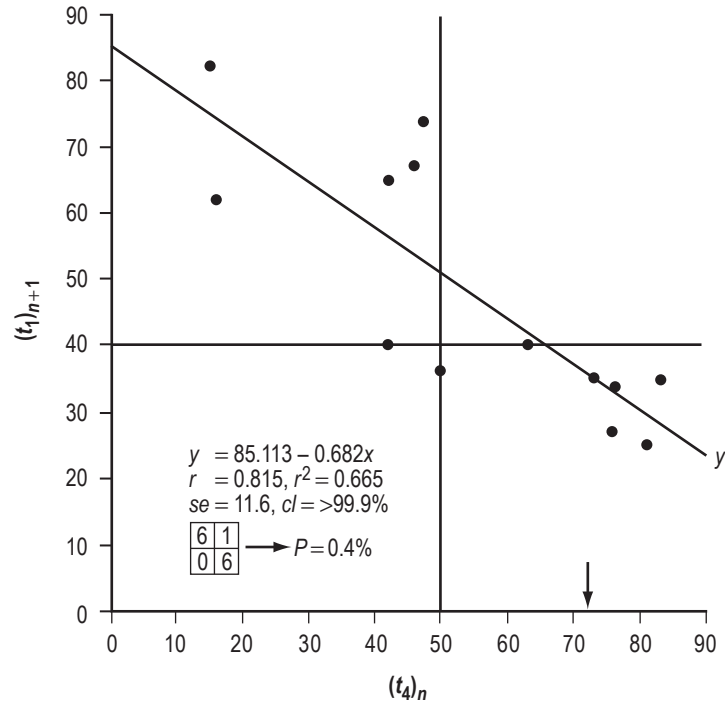


Figure 13. Scatterplot of t_1 for cycle $n + 1$ versus t_4 for cycle n .

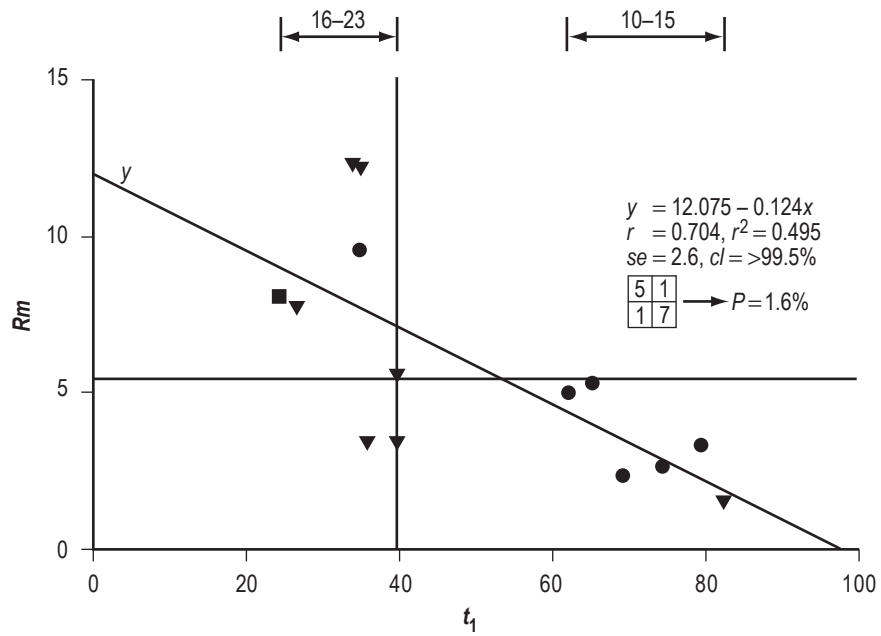


Figure 14. Scatterplot of Rm versus t_1 .

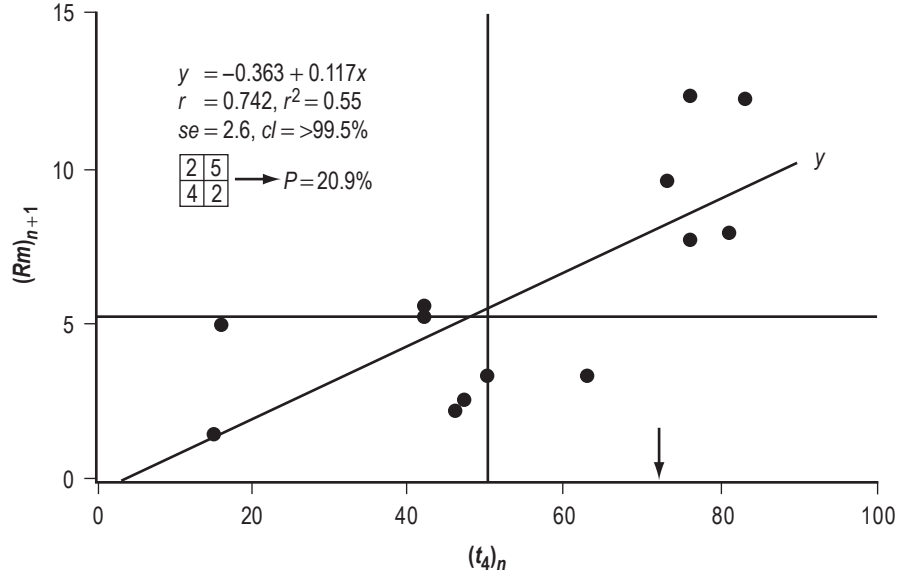


Figure 15. Scatterplot of Rm for cycle $n+1$ versus t_4 for cycle n .

Figure 16 shows the scatterplot of Rm against NSD during t_1 . The correlation is statistically important, whether expressed as a linear fit or in terms of a 2×2 contingency table. As noted earlier, NSD during t_1 has had a much narrower range during cycles 16–23 than during previous cycles (see fig. 8), averaging ≈ 198 days with a standard deviation of 78 days. Presuming that NSD during t_1 for cycle 24 will also be near 200 days, then, according to the linear fit, Rm for cycle 24 should be equal to $\approx 7.3 \pm 4.8$ —the 90-percent prediction interval. Based on the 2×2 contingency table, Rm should fall within the upper left quadrant, meaning that it should be larger than 5.1.

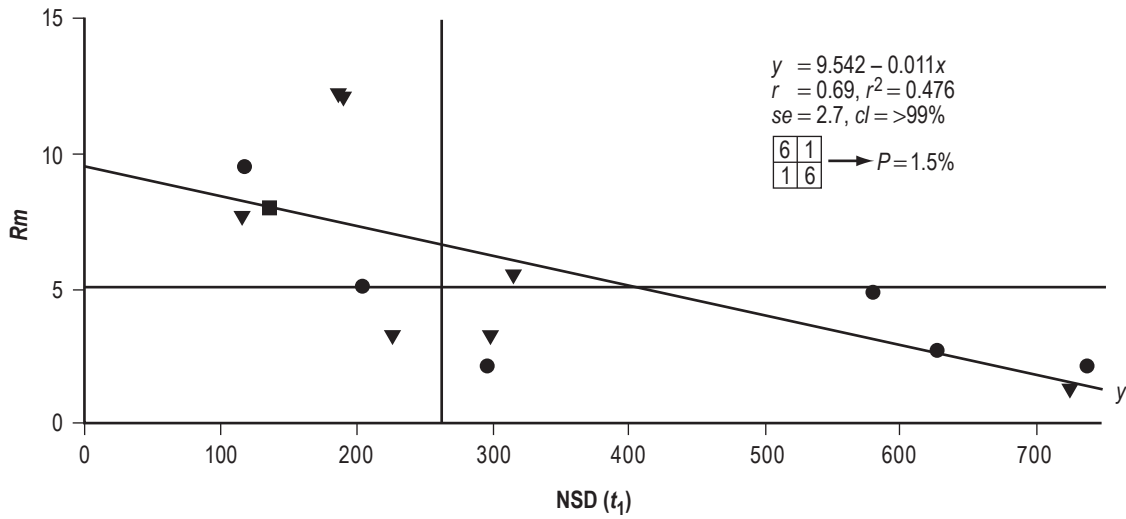


Figure 16. Scatterplot of Rm versus NSD during t_1 .

Figure 17 displays scatterplots of RM against t_1 , t_2 , and t_3 . Expressed as linear fits, all correlations are found to be statistically important. Expressed as 2×2 contingency tables, only the correlation between RM and t_3 is found to be statistically important; the correlation between RM and t_1 is marginally significant. As yet, none of these parameters is strictly known, although an estimate for the expected value of t_1 for cycle 24 has been given above (from fig. 13 on the basis of the value of t_4 for cycle 23) and values for all the parameters can be estimated from their cyclic variations (fig. 5). Because t_1 for cycle 24 is expected to be <40 mo, probably in the range of 25 to 36 mo, using the average value of t_1 for cycles 16–23 (equal to 34 mo), implies that RM for cycle 24 is expected to be equal to $\approx 136.4 \pm 61.8$ —the 90-percent prediction interval. For t_2 , using the average value found for cycles 16–23 (equal to 25 mo) suggests an RM equal to $\approx 133.6 \pm 59.9$ —the 90-percent prediction interval. Similarly, for t_3 , using its average value for cycles 16–23 (equal to 54 mo) suggests an RM equal to $\approx 139.500 \pm 54.5$ —the 90-percent prediction interval. Together, these predictions seem to support the view that RM for cycle 24 likely will be larger than average in size, hence, located within the upper left quadrant of each of the subfigures.

Figure 18 depicts scatterplots of RM against NSD during t_1 and NSD during ASC. For the RM against NSD scatterplots, the correlation is statistically significant, whether plotted linearly or as a 2×2 contingency table. Using a value of ≈ 200 for NSD (t_1), RM for cycle 24 is expected to be $\approx 135.3 \pm 55.6$ —the 90-percent prediction interval, based on the linear fit, and greater than or equal to ≈ 115 , based on the 2×2 contingency table. For the latter, neither the linear fit nor the 2×2 contingency table is found to be statistically important, only of marginal or near marginal significance.

Figure 19 shows the scatterplot of PER versus $t_3 + t_4$. On the basis of the 2×2 contingency table, the distribution appears to be of marginal statistical significance. The importance of the plot is that a value of 117 (see table 1) for $t_3 + t_4$ has been determined for cycle 23, denoted by the downward-pointing arrow along the x axis, inferring that cycle 23 likely will be a cycle of shorter period. Hence, its period should fall in the lower left portion of the figure.

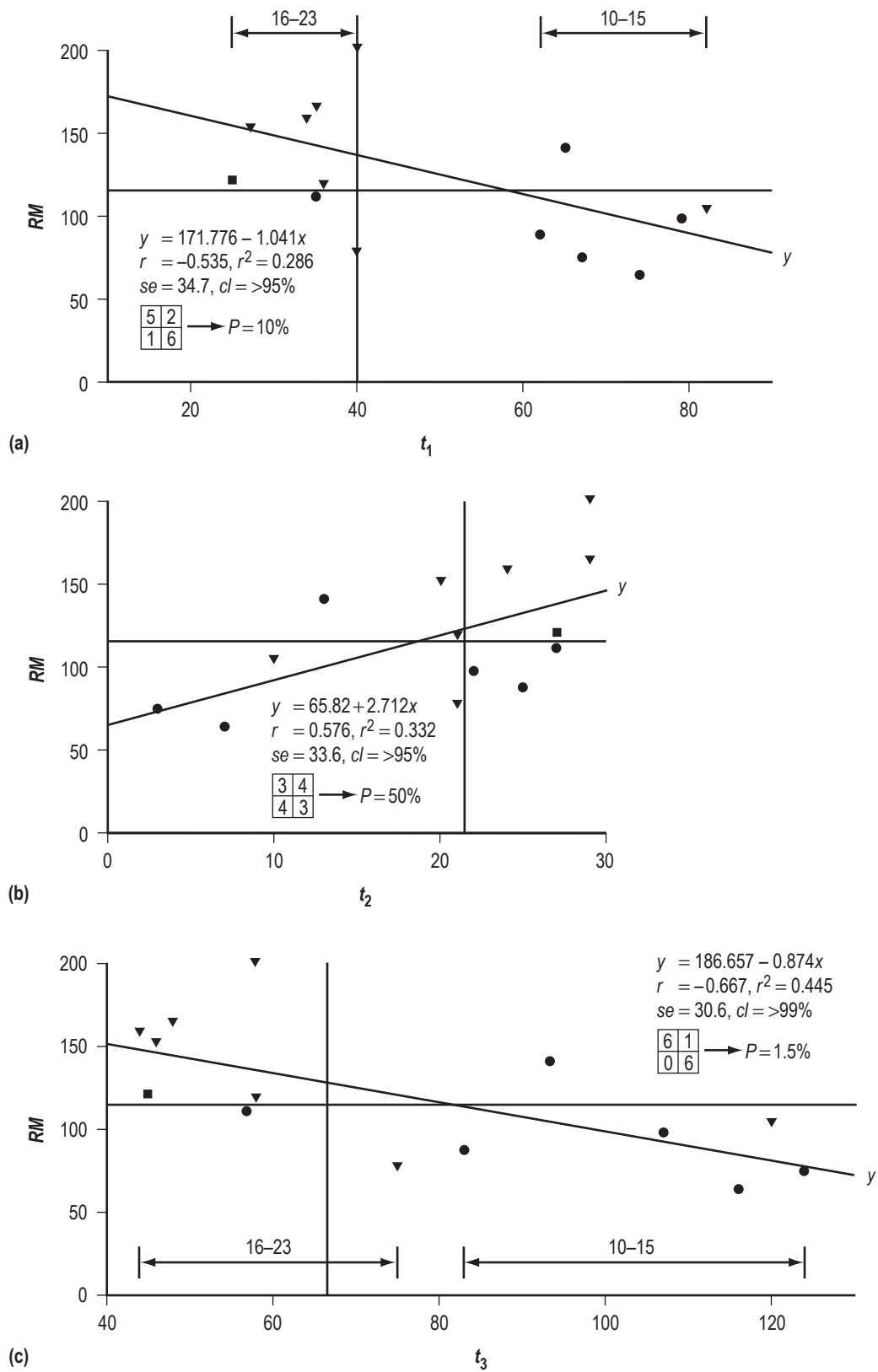


Figure 17. Scatterplots of RM versus (a) t_1 , (b) t_2 , and (c) t_3 .

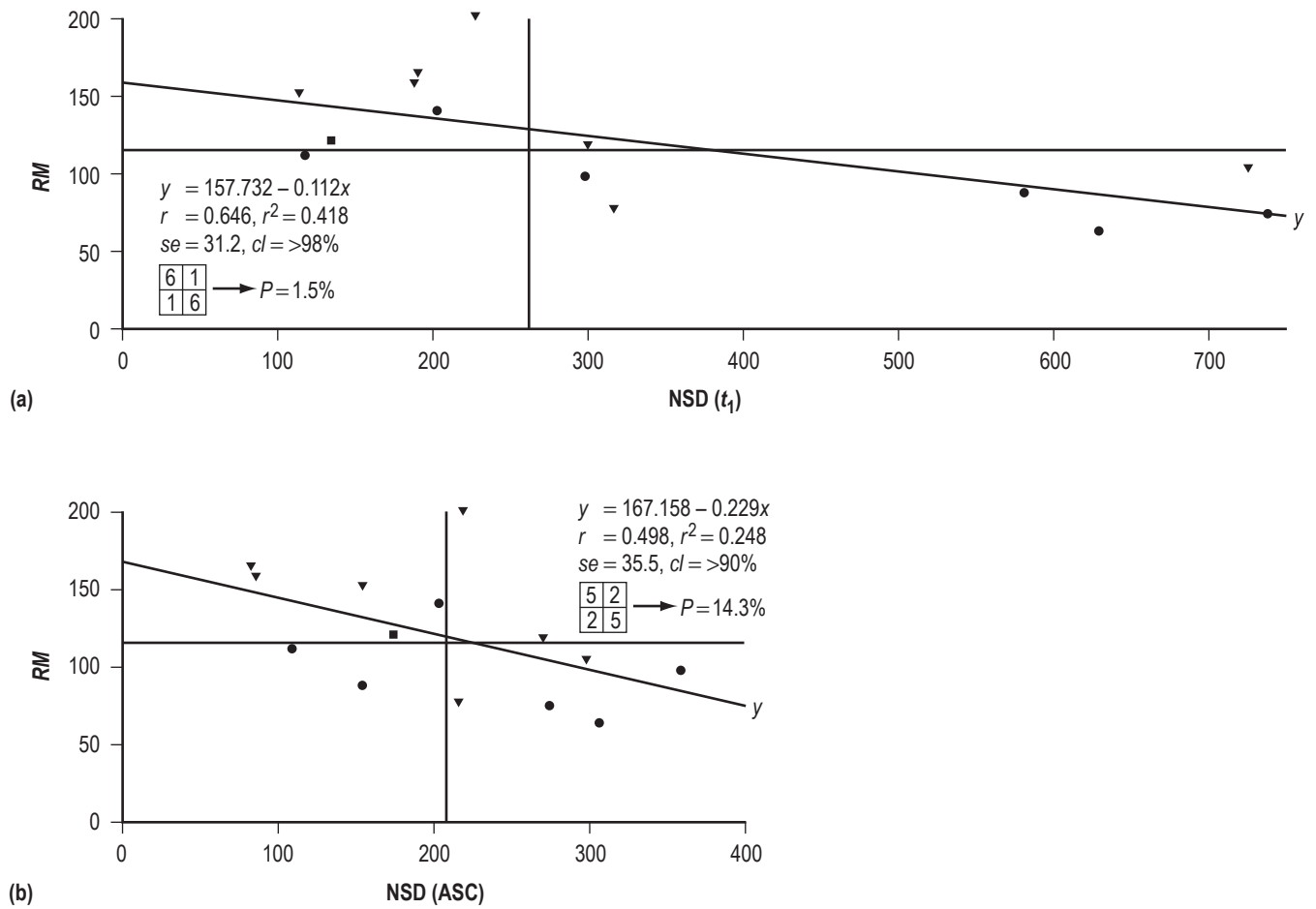


Figure 18. Scatterplots of RM versus (a) NSD during t_1 and (b) NSD during ASC.

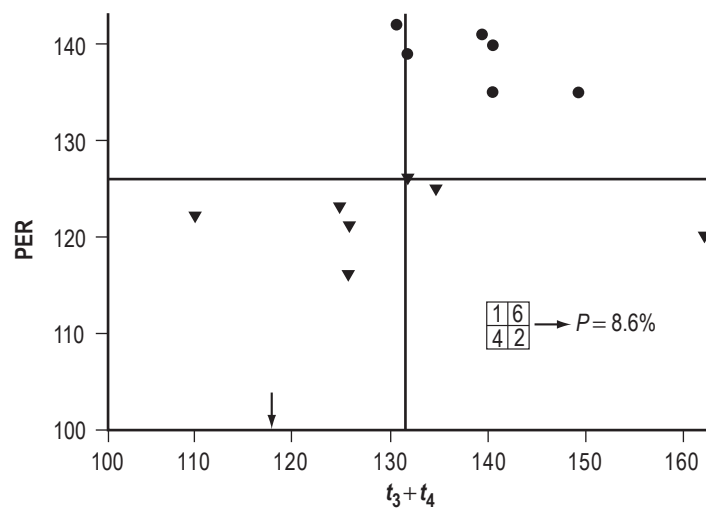


Figure 19. Scatterplot of PER versus $t_3 + t_4$.

3. CONCLUSION

Section 2 has shown that spotless days can be used to characterize the sunspot cycle, in particular, the timing and size of its minimum and maximum amplitudes, and perhaps the length of the cycle. Variations of possibly systematic behavior are seen in the time between first spotless day occurrence and minimum amplitude occurrence (t_1), the time between last spotless day occurrence and maximum amplitude occurrence (t_2), the time between first and last spotless day occurrences (t_3), the time between last spotless day occurrence for cycle n and first spotless day occurrence for cycle $n+1$ (t_4), the time between maximum amplitude occurrence for cycle n and first spotless day occurrence for cycle $n+1$ (mathematically equivalent to $t_4 - t_2$) and the time between first spotless day for cycle's n and $n+1$ (or $t_3 + t_4$). For cycles 16–23, t_1 has averaged ≈ 34 mo, ranging between 25 and 40 mo; t_2 has averaged ≈ 25 mo, ranging between 20 and 29 mo; t_3 has averaged ≈ 54 mo, ranging between 44 and 75 mo; and t_4 has averaged ≈ 72 mo, ranging between 50 and 83 mo. Also, a statistically significant, linear downward trend ($r = -0.7$ and $se = 63.8$ days) in actual number of spotless days occurring in the ascending portion of the cycle is suggested, such that during the rise of cycle 24 there is only a 5-percent chance that more than 215 spotless days are expected. Likewise, there appears to exist a statistically significant, linear upward trend ($r = 0.726$ and $se = 2.6$ units of sunspot number) in Rm , such that for cycle 24 there is only a 5-percent chance that it will measure < 5.9 .

Parameters t_2 , t_3 , and t_4 are each correlated with t_1 and t_1 for cycle $n+1$ correlates strongly with t_4 for cycle n . Because E(FSD) for cycle 24 occurred in January 2004, t_4 for cycle 23 measures ≈ 72 mo. Such a value suggests that t_1 for cycle 24 will measure about 25–36 mo, implying that minimum amplitude for the next cycle will occur sometime in 2006 and that cycle 23 is a cycle of shorter period (also true from the PER versus $t_3 + t_4$ plot; see fig. 19).

The maximum amplitude RM is found to correlate strongly with t_1 . Presuming that t_1 for cycle 24 will be about 25–36 mo, RM for cycle 24 is expected to be ≈ 136 , which agrees closely with its expected value based on the inferred linear upward trend of maximum amplitude of group sunspot number.

Figure 20 shows the variation relative to sunspot minimum occurrence of the average number of spotless days and the number of cycles having spotless days for cycles 16–23. Clearly, the nearer to E_m , the larger the average number of spotless days per month, with more than half of cycles 16–23 having a spotless day within 2 yr of sunspot minimum. If cycle 24's minimum is to occur in the latter half of 2006, then plainly the number of spotless days will become more pronounced during 2005 and 2006.

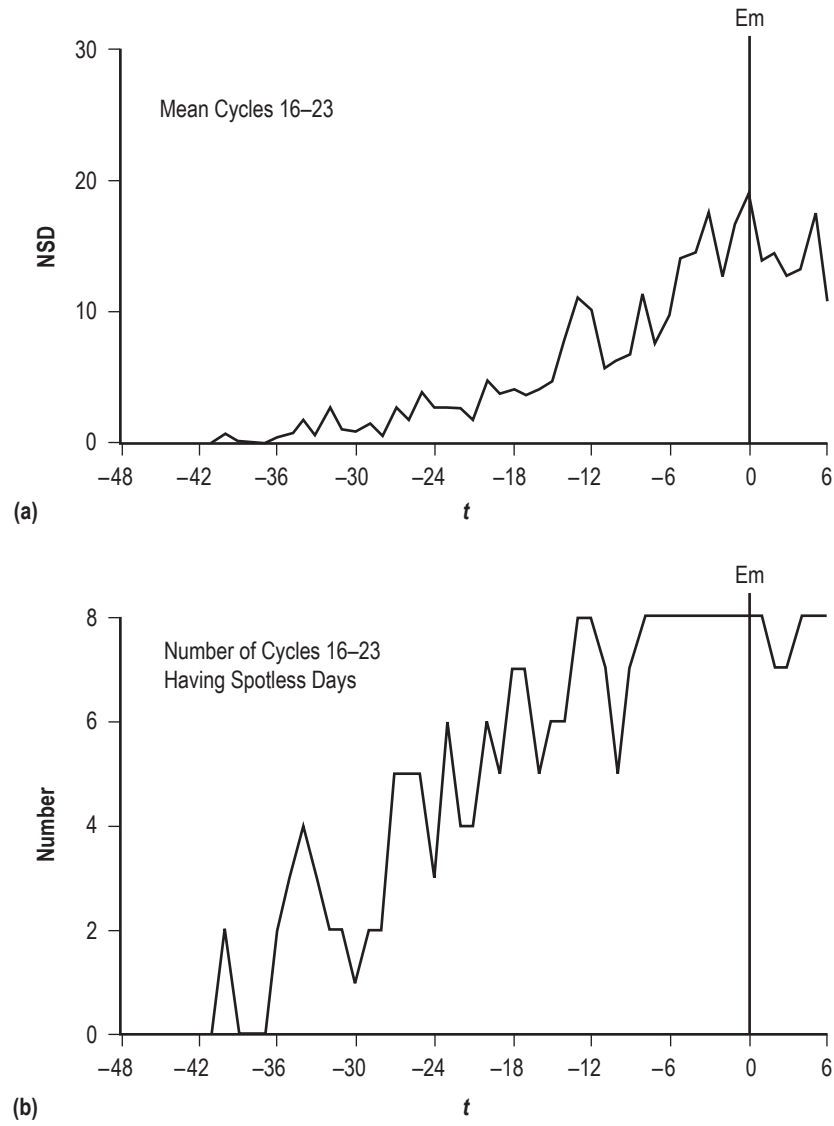


Figure 20. Variation of the (a) average number of spotless days and (b) number of cycles having spotless days for cycles 16–23 between t equal to –48 and 6 mo relative to Em.

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